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Fermion scattering in domain walls with a locally dependent phase.

E. Torrente Lujan.
Inst. fur Theoretische Physik, Universitat Bern
Sidlerstrasse 5, 3012 Bern, Switzerland.
e-mail: e.torrente@cern.ch

Abstract

We consider interactions of fermions with the domain wall bubbles produced during a first order phase transition. A new exact solution of the Dirac equations is obtained for a wall profile incorporating a position dependent phase factor. The reflection coefficients are obtained.

PACS:

1 Introduction

During the last few years, a considerable amount of work has been dedicated to the possibility of generating a sizeable baryon asymmetry on the electro-weak phase transition (see for example [1, 2, 3, 4]). For an excess of baryons to develop in an Universe which initially has zero baryon number, the already well known following conditions, first enucianted by Sakharov, must be met: 1) Some interaction of elementary particles must violate baryon number. 2) C and CP must be violated in order that there is not a perfect equality between rates of $\Delta B \neq 0$ processes, since otherwise no asymmetry could evolve from an initially symmetric state. 3) A departure from thermal equilibrium must play an essential role, since otherwise CPT would assure compensation between processes increasing and decreasing the baryon number. Remarkably, the standard model of weak interactions may provide all the necessary ingredients for baryogenesis. In particular the third condition can be met if the weak phase transition is at least weakly first order.

In a first order phase transition, the conversion from one phase to the other occurs trough nucleation. This happens when the system is either supercooled or superheated. Bubbles of the "true" phase (with an expectation value of some Higgs field $v \neq 0$) expand rapidly absorbing the region of the "false" phase ($v \neq 0$). At the bubble surface there is a region or "wall", in principle of microscopic dimensions, which separates the phases. The speed of the expanding bubble walls could be in the range $0.1 - 0.9\ c$ [5]. Particles in the "false" (higher temperature) phase are reflected off the advancing bubble walls, while most particles in the low temperature phase are unable to catch up with the receding walls, and cannot equilibrate across the phase boundary. Thus one has a departure from equilibrium and a baryon asymmetry can be generated.

In the physical conditions of the early Universe the fermions moving through the bubble wall will interact also with the particles in the surrounding plasma, thus a full transport problem must be considered. A useful simplifying assumption is to decompose the process into two steps, one describing the production of the CP asymmetry on the transmission/reflection coefficients when the quarks/antiquarks are scattered on the wall, the second describing the transport and the eventual transformation of the CP asymmetry into a baryon asymmetry via the baryon number anomaly.

Assuming that the scattering from the wall is little affected by diffusion corrections, the effects of the surrounding plasma can be partially incorporated by introducing a Higgs field effective potential taking into account finite temperature corrections to the tree-level potential. The structure of the wall depends on this effective potential on a complicated way. Fermions passing through the domain wall acquire a mass which is proportional to the vacuum expectation value (VEV) of the Higgs field, which is determined from the equations of motion of the finite temperature effective action of the bubble. The problem of computation of transmission coefficients reduces to the solution of a Dirac equation with a space dependent mass term. Exact solutions has been obtained only for two simple cases: for a wall profile approximated by a step function ([4, 6]) and for an average Higgs

field profile of the type ([1]):

$$\phi(z) = \frac{v}{2}\left(1 + \tanh\left(\frac{z}{\delta}\right)\right) \tag{1}$$

where the width of the wall is given by

$$\delta = \frac{2\sqrt{2}}{v\sqrt{\lambda}} = \frac{\sqrt{2}}{M_H}$$

and $M_H = \sqrt{\lambda}v/2$ is the Higgs mass. This profile is the analytic solution of the Higgs field equation for a bubble with its normal along the z-axis and position at $z = 0$ under some simplifying assumptions. There is not variation of the complex phase of the Higgs field through the wall. Numerical solutions have been obtained for some more complicated profiles which incorporate locally dependent complex phases and which are considered to be "reasonable" enough although not neccesarily solution of any equation of motion (for example in [2]).

For a given wall profile, parallel to the x-y plane and normal to the z-axis in its own rest frame, in order to compute the reflection coefficient one need only the plane wave solutions of the Dirac equation for particles moving along the z-axis. For any other incoming direction, the problem can be reduced to the latter performing an appropriate Lorentz boost. Following [1, 7], it is advantageous to work in the chiral basis, reordering the spinorial components the Dirac operator can be factorized into 2×2 blocks. For solutions with positive energy E

$$\Psi = e^{-iEt}\begin{pmatrix} \psi_I(z) \\ \psi_{II}(z) \end{pmatrix} \equiv e^{-iEt}\begin{pmatrix} \psi_1 \\ \psi_3 \\ \psi_4 \\ \psi_2 \end{pmatrix} \tag{2}$$

where ψ_1 and ψ_2 are eigenspinors of the chirality operator , γ_5, for the eigenvalue $+1$ and ψ_3, ψ_4 for -1, we obtain the two equations

$$(i\partial_z + Q(z))\psi_I = 0 \tag{3}$$

$$(i\partial_z + Q^*(z))\psi_{II} = 0 \tag{4}$$

With

$$Q(z) = \begin{pmatrix} E & -m(z) \\ m(z)^* & -E \end{pmatrix} \tag{5}$$

We will deal in this work with the particular case given by the function

$$m(z) = \begin{cases} m_0 \exp i(-\Delta\theta\lambda z + \theta_0) \exp -\lambda z & \text{if } z > 0 \\ m_0 \exp i\theta_0 & \text{if } z < 0 \end{cases} \tag{6}$$

All the constants are supposed real. $\lambda > 0$. This function represents a linear phase variation with a global difference of $\Delta\theta$ over a distance of the order of the wall thickness $\delta \equiv 1/\lambda$.

2 Solving the Dirac equation.

It would be possible to solve Eqs.(3-4) with the function given by Eq.(6), reducing them to a Bessel-like differential equation. It is possible and advantageous however to use perturbation theory to compute the evolution operator of the system. The summation to all orders of the perturbation expansion is possible thanks to the special form of $Q(z)$. There are two main advantages in doing so: the first one is that the procedure is easily generalizable to any number of dimensions (for example to incorporate mixing between generations), the second one is that the evolution operator is computed directly and the reflection coefficients are easily given in terms of its components. A similar technique has been used already in ([8, 9]) to compute the neutrino oscillation probabilities in solar matter.

The evolution operator of the differential system (3) is given by the path-ordered integral

$$U(z, z_0) = P \exp -i \int_{z_0}^{z} dz Q(z) \tag{7}$$

In this work we are concerned with an operator Q of the form

$$Q = Q^0 + V(z) \tag{8}$$

With

$$Q^0 = \begin{pmatrix} E & 0 \\ 0 & -E \end{pmatrix}; \quad V = \begin{pmatrix} 0 & -k \exp -\sigma z \\ k^* \exp -\sigma^* z & 0 \end{pmatrix} \tag{9}$$

k, σ are in general complex. The real part of σ is greater than zero. It can be set $\Re\sigma = 1$ without loss of generality.

Formally, it is possible to solve Eq.(7) by successive iterations:

$$U(z, z_0) = U^{(0)}(z, z_0) + \sum_{n=1}^{\infty} U^{(n)}(z, z_0); \quad U^{(0)}(z, z_0) = \exp\left(-iQ^0(z - z_0)\right) \tag{10}$$

$U^{(n)}$ is the well-known integral

$$U^{(n)} = (-i)^n \int_\Gamma dz_n \ldots dz_1 U^0(z, z_n) V(z_n) \ldots U^0(z_2, z_1) V(z_1) U^0(z_1, z_0) \tag{11}$$

The domain of integration is defined by

$$\Gamma \equiv z > z_n > \ldots > z_1 > z_0.$$

Following the same arguments as in [8] one can show that it is enough to compute U in the $z \to \infty$ limit. The evolution for finite time can be deduced from the expression for this limit. Trought elementary manipulations of Eq.(11), we get the elements of $U^{(n)}$ in a basis of eigenvectors of Q^0:

$$\begin{aligned} < b \mid U^{(n)} \mid a >= (-i)^n \exp(-i(Q^0_b z - Q^0_a z_0)) \times \\ \times \sum_{k1,\ldots,k(n-1)} \int_\Gamma d^n\tau \exp(iz_n w_{bk1} + \ldots + iz_1 w_{k(n-1)a}) V_{bk1}(z_n) \ldots V_{k(n-1)a}(z_1) \end{aligned} \tag{12}$$

With $w_{k1k2} = Q^0_{k1} - Q^0_{k2}$. Q^0_k one of the two eigenvalues of Q^0.

Due to the dimensionality of the problem and the special form for V, the summatory in Eq.(12) either is zero or reduces to only one term depending on whether n is odd or even and on the states a, b. For diagonal terms ($a = b$) the product of $V \ldots V$ is always zero when n is odd. When n is even there is a single surviving term. For non-diagonal terms, the situation is reversed: the only one surviving term appears for n odd. This single remaining term is always of the alternating form $\ldots V_{12}V_{21}V_{21}\ldots$. So,

$$< b \mid U^{(n)} \mid a >= (-i)^n \exp(-i(Q^0_b z - Q^0_a z_0)) I^{(n)}_{ab} \times \begin{cases} (- \mid k \mid)^{n/2} & a = b \\ V_{ba}\,(- \mid k \mid)^{(n-1)/2} & a \neq b \end{cases} \tag{13}$$

All the functions appearing in the integral $I^{(n)}_{ab}$ are of exponential type, the following equality ([8]) can be applied:

$$\begin{aligned} I_n(w_1, \ldots, w_n) &\equiv \int_{z_0}^{\infty} \ldots \int_{z_0}^{x_2} dx_n \ldots dx_1 \exp \sum_n w_n x_n \\ &= \frac{(-1)^n \exp(z_0 \sum_n w_n)}{w_1(w_1 + w_2)\ldots(w_1 + w_2 \ldots + w_n)} \\ &\quad (\text{valid if } \ \Re\, w_n < 0, \ \forall\ n\ ) \end{aligned} \tag{14}$$

In our case

$$\begin{aligned} w_1 + w_2 + \ldots + w_j &= iw_{bk1} - L + iw_{k1k2} - L^* + \ldots + iw_{k(j-1)kj} - L \\ &= iw_{bkj} + n_1 L + n_2 L^* \end{aligned} \tag{15}$$

$w_{bk(j)}$ can take only the values $\{0, \pm 2E\}$. L is σ or σ^*. One have for the integers n_1, n_2: $n_1 = n_2$ or $n_1 = n_2 \pm 1$.

For the diagonal terms, $a = b$, n even (writing $s = 2Ei - \sigma$):

$$\begin{aligned} I^{(n),even}_{aa} &= \frac{\exp z_0(n/2)(s+s^*)}{s^*(s^*+s)(s^*+s+s^*)\ldots((n/2)(s+s^*))} \\ &= \frac{\exp z_0(n/2)(s+s^*)}{\prod_{j=2,even}^{n}(j/2)(s+s^*)\prod_{j=1,odd}^{n-1}[s^* + (j-1)(s+s^*)/2]} \\ &= \frac{\exp z_0(n/2)(s+s^*)}{(s+s^*)^{n/2}\,(\frac{n}{2})!(s+s^*)^{n/2}\prod_{l=1}^{n/2}[s^*/(s+s^*) + (l-1)]} \\ &= \frac{\exp z_0(n/2)(s+s^*)}{(s+s^*)^n\,(\frac{n}{2})!\,[s^*/(s+s^*)]_{(n/2)}} \end{aligned} \tag{16}$$

For non-diagonal terms, taking $a = 1, b = 2$ to simplify the notation:

$$I^{(n),odd}_{ab} = \frac{(-1)\exp z_0((n/2)(s+s^*) + s^*)}{s^*(s^*+s)(s^*+s+s^*)\ldots((n/2)(s+s^*)+s^*)}$$

$$
\begin{aligned}
&= \frac{(-1)\exp z_0(n/2)(s+s^*)}{\prod_{j=2,even}^{n-1}(j/2)(s+s^*)\prod_{j=1,odd}^{n}(s^*+(j-1)(s+s^*)/2)} \\
&= \frac{(-1)\exp z_0(n/2)(s+s^*)}{(s+s^*)^{(n-1)/2}\left(\frac{n-1}{2}\right)!(s+s^*)^{(n+1)/2}\prod_{l=1}^{(n+1)/2}[s^*/(s+s^*)+(l-1)]} \\
&= \frac{(-1)\exp z_0(n/2)(s+s^*)}{(s+s^*)^n\left(\frac{n-1}{2}\right)!\,[s^*/(s+s^*)]_{(n+1)/2}} \qquad (17)
\end{aligned}
$$

We have used the Pochammer symbol defined by Eq. (31).
So, inserting Eqs.(16,17) in Eq.(13) and taking $s+s^*=-2$, $z_0=0$:

$$
\begin{aligned}
&< 1 \mid U \mid 1 >= \\
&= e^{-iEz}\left(1+\sum_{n=2,even}^{\infty}(-i)^n\left(-\mid k\mid^2\right)^{n/2}\frac{1}{(-2)^n(\frac{n}{2})![-s^*/2]_{(n/2)}}\right) \\
&= e^{-iEz}\left(1+\sum_{m=1}^{\infty}\left(\frac{\mid k\mid^2}{4}\right)^m\frac{1}{m![-s^*/2]_{(m)}}\right) \\
&= e^{-iEz}{}_0F_1\left(-\frac{s^*}{2};\frac{\mid k\mid^2}{4}\right) \qquad (18)
\end{aligned}
$$

$$
\begin{aligned}
&< 1 \mid U \mid 2 >= \\
&= e^{-iEz}(-k)\sum_{n=1,odd}^{\infty}(-i)^n\left(-\mid k\mid^2\right)^{(n-1)/2}\frac{(-1)}{(-2)^n(\frac{n-1}{2})![-s^*/2]_{((n+1)/2)}} \\
&= e^{-iEz}\frac{2ik}{\mid k\mid^2}\sum_{m=1}^{\infty}\left(\frac{\mid k\mid^2}{4}\right)^m\frac{1}{(m-1)![-s^*/2]_{(m)}} \\
&= e^{-iEz}\frac{ik}{s^*}{}_0F_1\left(1-\frac{s^*}{2};\frac{\mid k\mid^2}{4}\right) \qquad (19)
\end{aligned}
$$

and similarly for the other two matrix elements. The matrix U can be written as

$$
\begin{aligned}
U(z\to\infty,z_0) &= \exp -iH_0(z-z_0)U_{red}(z_0) \\
U_{red}(z_0) &= \begin{pmatrix} F & G \\ G^* & F^* \end{pmatrix} \qquad (20)
\end{aligned}
$$

with $\sigma=\lambda(1+i\Delta\theta)$, $k=-m_0\exp i\theta_0$,

$$
\begin{aligned}
G &= \frac{im_0\exp i\theta_0}{1+(2E-\Delta\theta)i}{}_0F_1\left(\frac{3}{2}+(E-\frac{\Delta\theta}{2})i;\frac{\mid m_0\mid^2}{4}\right) \\
F &= {}_0F_1\left(\frac{1}{2}+(E-\frac{\Delta\theta}{2})i;\frac{\mid m_0\mid^2}{4}\right) \qquad (21)
\end{aligned}
$$

In the case $z_0 \neq 0$, F, G would include a factor $\exp -2z_0$ in its argument. In these formulas E, m_0 are given in units of λ, the inverse of the wall thickness.

To obtain $\overline{U}$, evolution operator for the Eq.(4), one must make the changes $k \rightarrow k^*, \Delta\theta \rightarrow -\Delta\theta$. The matrix become

$$\begin{aligned} \overline{U}(z \rightarrow \infty, z_0) &= \exp -iH_0(z - z_0)\overline{U}_{red}(z_0) \\ \overline{U}_{red}(z_0) &= \begin{pmatrix} \overline{F} & \overline{G} \\ \overline{G}^* & \overline{F}^* \end{pmatrix} \end{aligned} \tag{22}$$

with

$$\begin{aligned} \overline{G} &= \frac{im_0 \exp -i\Delta\theta_0}{1 + (2E + \Delta\theta)i} {}_0F_1\left(\frac{3}{2} + (E + \frac{\Delta\theta}{2})i; \frac{\mid k \mid^2}{4}\right) \\ \overline{F} &= {}_0F_1\left(\frac{1}{2} + (E + \frac{\Delta\theta}{2})i; \frac{\mid k \mid^2}{4}\right) \end{aligned} \tag{23}$$

See Appendix A for some new formulas for the absolute values of generalized hypergeometric functions which can be obtained from the general properties of $U, \overline{U}$.

Following the same reasoning used in ([8]), using the general properties of the evolution operator, the evolution for any finite z is given by the matrix

$$U(z, z_0) \equiv U_s(z)^{-1}U_s(z_0) = U_{red}^{-1}(z)e^{-iQ^0(z-z_0)}U_{red}(z_0) \tag{24}$$

In this work, we will make use only of the infinite time limit.

3 The reflection Coefficient. Results.

We will follow the same procedure as in ([1]) for defining the reflection coefficient. In the region $z \rightarrow \infty$, the eigenspinors ψ_2, ψ_3 correspond to right-moving particles with chirality $\gamma_5 = +1, -1$ respectively. The solutions ψ_1, ψ_4 are identified with left-moving particles with chirality $+1, -1$. These states are also eigenstates of the hamiltonian in this region

The momentum eigenstates for $z < 0$ are obtained diagonalizing the constant matrix $Q(0)$ by

$$u(p) = \begin{pmatrix} \cosh\theta_p & \sinh\theta_p \\ \sinh\theta_p & \cosh\theta_p \end{pmatrix}, \quad \sinh 2\theta_p \equiv \frac{\mid m \mid}{p} = \frac{1}{\sqrt{(E/\mid m \mid)^2 - 1}} \tag{25}$$

with eigenvalues $p = \pm\sqrt{E^2 - \mid m \mid^2}$.

For left-moving particles incident from the symmetric phase, two components coexist at $z = \infty$, the incident particle itself and the reflected one by the domain wall. We define reflection coefficients $R, \overline{R}$ as

$$\psi_3(\infty) = R\psi_1(\infty); \quad \psi_2(\infty) = \overline{R}\psi_4(\infty) \tag{26}$$

Imposing the boundary condition that at $z < 0$ only a left-moving particle with momentum p propagates, one gets the expression:

$$\psi(\infty) = U(z \to \infty, 0)u^{-1}(p)\psi_p(0) \tag{27}$$

where ψ_p is a momentum eigenspinor. Then, the reflection coefficient is given by

$$R = \frac{(Uu^{-1})_{21}}{(Uu^{-1})_{11}}; \quad \overline{R} = \frac{\left(\overline{U}u^{-1}\right)_{21}}{\left(\overline{U}u^{-1}\right)_{11}} \tag{28}$$

or more explicitly:

$$R = \frac{U_{21} - tU_{22}}{U_{11} - tU_{12}} = e^{2Ezi}\frac{G^* - tF^*}{F - tG}; \quad t = \tanh\theta_p \tag{29}$$

and similarly for $\overline{R}$.

We are interested in the asymmetry between the reflection probabilities. The quantity $A =\mid R \mid^2 - \mid \overline{R} \mid^2$ is displayed in figs.(1-3) as a function of the diverse parameters involved $E, \Delta\theta, \theta_0$ and for different values of the product particle mass x wall width $\mid m \mid \delta$.

In Fig.(1) the asymmetry A is plotted as a function of the dimensionless quantity $E/\mid m \mid$ and some fixed values of $\Delta\theta$. For $\Delta\theta = \pi$ (plot B) the results obtained here coincide with the numerical result obtained in [1] (Fig. 2). As the phase difference $\Delta\theta$ increases, the height of the peaks keeps unaltered but their position moves according to the rule

$$(E/m)_{peak} =\mid \Delta\theta \mid /(2m\delta).$$

The first parameter of the hypergeometric functions in Eqs.(22-23) is real for these peak values. As the value of $\Delta\theta$ gets higher some resonant effect becomes apparent, the peaks get sharper and the asymmetry is only non-negligible around the peaks. This is particularly evident in the last plot (C). This effect could lead to an essentially monochromatic asymmetry in scenarios with highly oscillatory or random phase differences.

In Fig.(2) we check the influence of the value of the phase θ_0. The plot (A) corresponds to plot (C) in Fig.(1). Here we have set $\theta_0 = \pi$. Some new dips appear at lower E/m, otherwise the position and width of the peaks remain unaltered. In the plot (B) we present the variation of A (peak values at θ_0) in a full cycle of θ_0. The phase θ_0 appears only in the multiplicative exponential coefficient of G in Formula (21). From this, the soft, quasi-sinusoidal behavior observed in the graphic is understood.

Finally in Fig.(3) we plot A as a function of the parameter $\Delta\theta$, for θ_0 fixed and for different values of the product $m\delta$ as before. As expected the peaks in the asymmetry function appear for values such that

$$\Delta\theta_{peak} = 2E\delta.$$

As conclusion, we were able to solve the Dirac equation with a space dependent complex mass term which, although not a solution of the equations of motion, reproduces the

expected variation in module and in complex phase of the average Higgs field across the wall. From the solution of the Dirac equation the particle-antiparticle transmission asymmetry is computed. The analytical results presented here confirm previous numerical computations and predict an unexpected behavior for highly oscillatory phase fields.

We note that with little modifications our method can be used to solve the Dirac equation for a purely local dependent phase mass term ($\lambda \to 0, \Delta\theta\lambda \neq 0$ in Eq.(6) or equivalently σ purely imaginary in Eq.(9)). In order to circumvent convergence problems this must be done taking the limit $\Re\lambda \to 0$ in the finite propagation time Eq.(24).

A Appendix: some old and new formulas about Hypergeometric Functions

The generalized Hypergeometric function ([10]) is defined by

$$ {}_pF_q(a_1, \ldots, a_p, b_1, \ldots, b_q; z) = \sum_{n=0}^{\infty} \frac{(a_1)_{(n)} \cdots (a_p)_{(n)}}{(b_1)_{(n)} \cdots (b_q)_{(n)}} \frac{z^n}{n!} \tag{30} $$

where the Pochammer symbol is

$$ (z)_{(n)} = \Gamma(n+z)/\Gamma(z) \tag{31} $$

in particular

$$ {}_0F_1(b; z) = \sum_{n=0}^{\infty} \frac{1}{(b)_{(n)}} \frac{z^n}{n!} \tag{32} $$

The derivative of ${}_0F_1$ is again a ${}_0F_1$ function:

$$ \frac{d\ {}_0F_1(\gamma; z)}{dz} = \frac{1}{\gamma}\ {}_0F_1(1+\gamma; z) \tag{33} $$

The function ${}_0F_1$ is related to the Bessel Functions by the formula

$$ J_n(z) = \frac{(z/2)^n}{\Gamma(1+n)} {}_0F_1\left(1+n; -\frac{z^2}{4}\right) \tag{34} $$

For a Q as given by Eq.(8) is traceless, $det\ Q(z, z_0) = det\ Q(0,0) = 1$, or $\mid F \mid^2 - \mid G \mid^2 = 1$. The formulas (21) are valid for any $E, \Delta\theta$ real, k complex, we obtain:

$$ \mid {}_0F_1\left(\frac{1}{2} + \epsilon i; x^2\right) \mid^2 - \frac{x^2}{\mid 1/2 + \epsilon i \mid^2} \mid {}_0F_1\left(\frac{3}{2} + \epsilon i; x^2\right) \mid^2 = 1 \tag{35} $$

to be compared with the similar formulas obtained in ([8]) for the absolute values of the generalized hypergeometric functions ${}_nF_n$.

In fact, with little changes, one could compute U for any general matrix V with diagonal terms equal to zero and non-diagonal terms of general exponential type not neccesarily equal. In the particular case of V hermitic: $V_{12} = V_{21}^* = k$, U_{red} is unitary and of the form

$$U_{red}(z_0) = \begin{pmatrix} \overline{F} & \overline{G} \\ -\overline{G}^* & \overline{F}^* \end{pmatrix} \tag{36}$$

with $\overline{F}, \overline{G}$ given by Eq.(22).

By the unitarity of the matrix (36), $\mid F \mid^2 + \mid G \mid^2 = 1$. And we get the formula:

$$\mid J_{-\frac{1}{2}+\epsilon i}(x) \mid^2 + \mid J_{\frac{1}{2}+\epsilon i}(x) \mid^2 = \frac{2\cosh \pi\epsilon}{\pi x} \tag{37}$$

For $\epsilon = 0$ this formula reduces to the special case involving the well known Bessel functions of order 1/2: $J_{-1/2}(x) = \sqrt{2/(\pi x)}\cos(x)$; $J_{1/2}(x) = \sqrt{2/(\pi x)}\sin(x)$.

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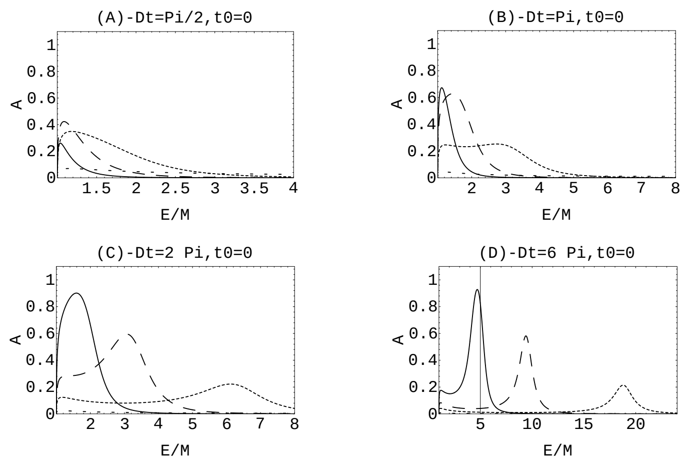

Figure 1: The asymmetry A as a function of $E/\mid m\mid$. Continuos line: $\mid m\mid\delta=2$, dashed lines: respectively $\mid m\mid\delta=1,1/2,1/10$. From A to C: $\Delta\theta=\pi/2,\pi,2\pi,6\pi$. For all figures $\theta_0=0$.

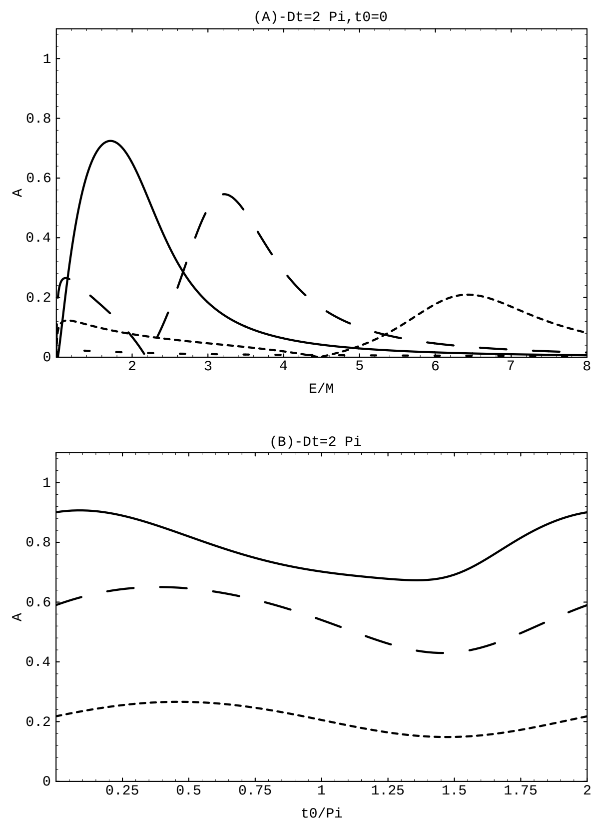

Figure 2: Dependence with the initial angle θ_0. Top figure: as in Fig.(1) (C) but here for $\theta_0 = \pi$. Bottom figure: the quantity A as a function of θ_0. $\Delta\theta = 2\pi$, $E/m = \Delta\theta/(2m\delta)$ (peaks in Fig.(1)(C)).

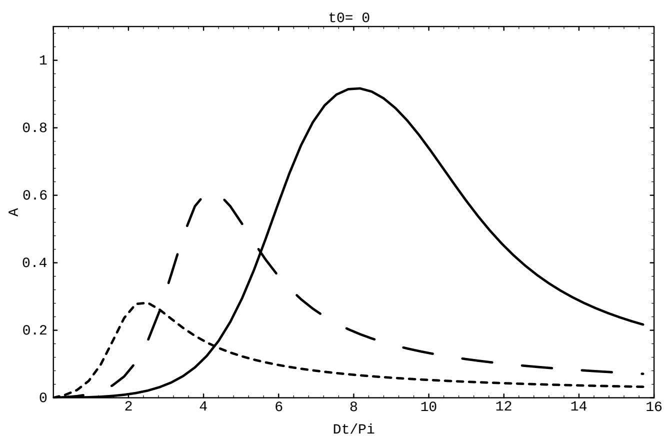

Figure 3: Asymmetry A as a function of $\Delta\theta$ and different $\mid m \mid \delta$ as before. $\theta_0 = 0$; $E\delta = 2m\delta$.